```
Lemma (prj):

  forall
    l ∈ L
  it holds that
    prj_l^M : M -> {cn | lev(c) = l}^●.

Proof.

  Pick l.
  Pick m.

  We show that
    prj_l^M : M -> {cn | lev(c) = l}^●.

  By definition of prj_l^M, either
    (prj_l^M m) = Nothing, or
    (prj_l^M m) = Just cn  for some c and n such that
    lev(c) = l.

  Thus
    (prj_l^M m) ∈ {cn | lev(c) = l}^●.

Qed.


Lemma (IProc-Stream).

  forall
    p ∈ IProc I O
    s,
  if
    p --s-►,
  then
    s ∈ Stream I O.

Proof.

  Follows from the definition of
  IProc I O,
  --s-►, and
  Stream I O.

Qed.


Theorem:

  forall
    l0 ∈ L
    p0 ∈ IProc M M^●.
  it holds that
    SE l0 p0 ∈ NI(=M,=M^●).

Proof.

  Pick l0 and p0.

  Pick s0 such that
    SE l p0 --s0-►.

  To show:
  forall l,
  there exists a relation
    R
  such that
    ⟨s0,SE l0 p0⟩ ∈ R
  and
```

```
  R is a l-(=M)-(=M^●)-stream-simulation.

***

Case on l.

***

Case not (l0 ⊑ l):

  Pick
    R = { ⟨s,SE l0 p⟩ | s ∈ Stream M {cn | lev(c) = l0}^● }.

  ***

  To prove:
    ⟨s0,SE l0 p0⟩ ∈ R

  Set
    s = s0, and
    p = p0.

  By Lemma (prj), and by definition of map,
    p ∈ IProc  M {cn | lev(c) = l0}^● ).

  By Lemma (IProc-Stream),
    s ∈ Stream M ( {cn | lev(c) = l0}^● ).

  Thus
    ⟨s,SE l0 p⟩ ∈ R.

  Thus
    ⟨s0,SE l0 p0⟩ ∈ R.

  ***

  To prove:
    R is a stream simulation.

  We prove that
  R satisfies 1), 2) and 3) in Def IV.2.

  ***

  case 1):

    Pick
      ⟨?m.s,SE l0 p⟩ ∈ R.

    ***

    To show:
      ⟨s,SE l0 p⟩ ∈ R.

    ***

    Since
      ⟨?m.s,SE l0 p⟩ ∈ R,
    we get
      ?m.s ∈ Stream M {cn | lev(c) = l0}^●.

    Thus,
      s ∈ Stream M {cn | lev(c) = l0}^●.

    Thus, by definition of s,
      ⟨s,SE l0 p⟩ ∈ R.

    thus R satisfies 1) to be a simulation.
```

```
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  ***

case 2):

  Pick
    ⟨s,SE l0 p⟩ ∈ R.

  ***

  To show:
  forall
    m =Ml ●,
  there exists
    pM
  such that
    (SE l0 p) ~~?m~▶ pM, and
    ⟨s,pM⟩ ∈ R.

  ***

  Pick
    m
  such that
    m =Ml ●.

  ***

  Since
    p is interactive,
  we get that
    p is input total.

  Since
    p is input total,
  we get that there is some
    p'
  such that
    p ~~?(obs_l^M m)~▶ p'.

  By definition of SE,
    (SE l0 p) ~~?m~▶ (SE l0 p').

  ***

  Set
    pM = (SE l0 p').

  ***

  Thus, by definition of s and pM,
    ⟨s,pM⟩ ∈ R.

  thus R satisfies 1) to be a simulation.

  ***
  ***
  ***

Case 3):

  Pick
    ⟨?m.s,SE l0 p⟩ ∈ R.

  ***

  To show:
  forall
```

```
      m' =Ml m,
    there exists
      pM
    such that
      (SE l0 p) ~~?m'~▶ pM, and
      ⟨s,pM⟩ ∈ R.

    ***

    Pick
      m'
    such that
      m' =Ml m.

    ***

    Since
      p is interactive,
    we get that
      p is input total.

    Since
      p is input total,
    we get that there is some
      p'
    such that
      p ~~?(obs_l^M m')~▶ p'.

    By definition of SE,
      (SE l0 p) ~~?m'~▶ (SE l0 p').

    ***

    Set
      pM = (SE l0 p').

    ***

    Since
      ?m.s ∈ Stream M ( {cn | lev(c) = l0}^● ),
    we have that
      s ∈ Stream M ( {cn | lev(c) = l0}^● ).

    ***

    Thus, by definition of s and pM,
      ⟨s,pM⟩ ∈ R.
    thus R satisfies 2) to be a simulation.

    ***
    ***
    ***

  Case 4):

    Pick
      ⟨!m.s,(SE l0 p)⟩ ∈ R.

    Since
      !m.s ∈ Stream M ( {cn | lev(c) = l0}^● ), and
      not (l0 ⊑ l),

    we get that
      m =Ml ●.

    ***

    To show:
    there exists
```

```
    m', and
    pM
  such that
    m' =Ml m
    (SE l0 p) ——!m'—▶ pM, and
    ⟨s,pM⟩ ∈ R.

  ***

  Since
    p is interactive,
  we get that
    p is output productive.

  Since
    p is output productive,
  we get that there is some
    mP, and
    p'
  such that
    p ——!mP—▶ p'.

  ***

  Set
    m' = prj_l0^M mP.

  Then
    (SE l0 p) ——!m'—▶ (SE l0 p').

  Set
    pM = (SE l0 p').

  ***

  Since
    not (l0 ⊑ l),
  we get by definition of prj_l0^M that
    m' =Ml ●.

  By transitivity of (=Ml), we get that
    m' =_l m.

  ***

  Since
    !m.s ∈ Stream M ( {cn | lev(c) = l0}^● ),
  we have that
    s ∈ Stream M ( {cn | lev(c) = l0}^● )

  ***

  Thus, by definition of s and pM,
    ⟨s,pM⟩ ∈ R.

  thus R satisfies 3) to be a simulation.

  ***
  ***
  ***

  thus R is a simulation.

  ***
  ***
  ***

case (l0 ⊑ l).
```

```
Pick
  R = { <s,SE l0 p> | SE l p --s0-▶ }.

***

To prove:
  ⟨s0,SE l p⟩ ∈ R

Set
  s  = s0,
  p  = p0.

Then
  ⟨s,SE l0 p⟩ ∈ R.

Thus,
  ⟨s0,SE l p⟩ ∈ R.

***
***
***

To prove:
  R is a stream simulation.

We prove that
R satisfies 1), through 4) in Def IV.2.

***

case 1):

  Pick
    ⟨?m.s,(SE l0 p)⟩ ∈ R
  such that
    m =Ml ●.

  To show:
    ⟨s,(SE l0 p)⟩ ∈ R.

  ***

  By definition of R,
    (SE l0 p) --?m.s-▶ω.

  ***

  By definition of (=Ml), since
    m =Ml ●,
  we get that
    m = cn
  for some c for which not(lev(c) ⊑ l).

  since
    l0 ⊑ l,
  we get
    not(lev(c) ⊑ l0).
  Thus,
    obs_l0^M m = Nothing.

  ***

  By definition of SE, and since p is interactive (input concrete),
    (SE l0 p) ~~?m~▶ (SE l0 p).

  Since
    (SE l0 p) --?m.s-▶ω,
  we get
    (SE l0 p) ~~?m~▶ (SE l0 p) --s-▶ω,
```

```
  and thus,
    (SE l0 p) --s-▶ω.

  ***

  Set
    pM = (SE l0 p).

  Since
    ⟨s,(SE l0 p)⟩ ∈ R,
  we get by definition of pM that
    ⟨s,pM⟩ ∈ R.

  ***
  ***
  ***

case 2):

  Pick
    ⟨s,(SE l0 p)⟩ ∈ R

  To show:
  forall
    m =Ml ●
  there exists
    pM
  such that
    (SE l0 p) ~~?m~▶ pM, and
    ⟨s,pM⟩ ∈ R.

  ***

  Pick
    m
  such that
    m =Ml ●.

  ***

  By definition of (=Ml), since
    m =Ml ●,
  we get that either
    m = ●, or
    m = cn
  for some c for which not(lev(c) ⊑ l).

  In the latter case, since
    l0 ⊑ l,
  we get
    not(lev(c) ⊑ l0).
  Thus, for both cases of m,
    obs_l0^M m = ●.

  ***

  By definition of SE, and since p is interactive (input concrete),
    (SE l0 p) ~~?m~▶ (SE l0 p).

  ***

  Set
    pM = (SE l0 p).

  Since
    ⟨s,(SE l0 p)⟩ ∈ R,
  we get by definition of pM that
    ⟨s,pM⟩ ∈ R.
```

```
  ***
  ***
  ***

case 3):

  Pick
    ⟨?m.s,(SE l0 p)⟩ ∈ R

  To show:
  for all
    m' =Ml m
  there exists
    pM
  such that
    (SE l0 p) ~~?m'~▶ pM, and
    ⟨s,pM⟩ ∈ R.

  ***

  By definition of R,
    (SE l0 p) --?m.s-▶ω.
  By definition of SE and ---▶ω (and (MAP_IN●), (MAP_IN)),
  there is some
    p'
  for which
    (SE l0 p) ~~?m~▶ (SE l0 p') --s-▶, and
    p ~~?(obs_l0^M m)~▶ p'.

  ***

  Pick
    m'
  such that
    m' =Ml m.

  ***

  Case on m.

  ***

    case m =Ml ●:

      Since

        l0 ⊑ l,

      we get

        obs_l0^M m = ●.

      ***

      Since

        m' =Ml m.

      we get by definition of (=Ml) that either

        m' = ●, or
        m' = cn,

      for some c for which not(lev(c) ⊑ l).
      In the latter case, since

        l0 ⊑ l,
```

```
  we get

    not(lev(c) ⊑ l0).

  Thus, for both cases of m',

    obs_l0^M m' = ●.

  ***

  Since

    obs_l0^M m  = ●,
    obs_l0^M m' = ●, and
    p ~~?(obs_l0^M m)~▶ p',

  we get

    p ~~?(obs_l0^M m')~▶ p'.

  Thus

    (SE l0 p) ~~?m'~▶ (SE l0 p') --s-▶.

  ***

  Set

    pM = (SE l0 p').

  Then, by definition of s and pM,

    ⟨s,pM⟩ ∈ R.

  ***
  ***
  ***

case not(m =Ml ●):

  Then

    m = cn,

  for some c for which (lev(c) ⊑ l).

  By definition of obs_l0^M, since

    m =Ml m',

  we get

    m' = m.

  ***

  Thus, since

    p ~~?(obs_l0^M m)~▶ p', and
    (SE l0 p) ~~?m~▶ (SE l0 p') --s-▶,

  we get

    p ~~?(obs_l0^M m')~▶ p', and
    (SE l0 p) ~~?m'~▶ (SE l0 p') --s-▶.

  ***

  Set
```

```
        pM = (SE l0 p').

      Then, by definition of s and pM,

        ⟨s,pM⟩ ∈ R.

      ***
      ***
      ***

  thus R satisfies 2) to be a simulation.

  ***
  ***
  ***

case 4):

  Pick

    ⟨!m.s,(SE l p)⟩ ∈ R

  To show:
  there exists

    m'

  such that

    m' =Ml m,
    (SE l0 p) ——!m'—▶ pM, and
    ⟨s,pM'⟩ ∈ R.

  ***

  By definition of R,

    (SE l0 p) --!m.s-▶ω.

  By definition of SE and ---▶ω (and (MAP_OUT)),
  there is some

    p'

  for which

    (SE l p) ——!m—▶ (SE l p') --s-▶.

  By definition of R,

    ⟨s,(SE l p')⟩ ∈ R.

  ***

  Set

    m'  = m
    pA' = (SE l p').

  Then

    ⟨s,pA'⟩ ∈ R.

  ***

  thus R satisfies 4) to be a simulation.

***
```

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      ***
      ***

      thus R satisfies 1) to be a simulation.

      ***
      ***
      ***

    Thus R is a l-(=M)-(=M)-stream-simulation.

  Thus,
  for all l,
  there exists a relation

    R

  such that

    ⟨s0,SE l0 p0⟩ ∈ R

  and

    R is a l-(=M)-(=M)-stream-simulation.

Qed.
```